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Contact problem of elastic strips with initial stresses with periodically placed resilient protective strap

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Abstract

The present article considers the formulation and solution of the problem on contact interaction of a pre-stressed band with regularly placed elastic cover plates. The study has been carried out within the framework of the linearized elasticity theory in general form for the theory of large (finite) initial deformations and two variants of the theory of small initial deformations with an arbitrary structure of the elastic potential [1]. The solution of the problem comes down to a singular integro-differential equation for unknown contact stresses with a kernel expressed in the form of a sum of the kernel and some regular kernel under certain boundary conditions. The solution sought is given in the form of a series of Jacobi polynomials. A quasiregular infinite system of linear equations is obtained in order to determine the unknown coefficients of the series [2].

Key words: linearized theory of elasticity, initial (residual) stress, contact problems, Fourier integral transformations.

Introduction

Contact interaction of initially stressed bodies is one of topical issues in the field of mechanics of deformable solids. The solution to such problems is related to a wide range of questions that arise in engineering, construction and other industries. Strict conditions have been formulated recently: high (guaranteed) durability of structures and machine parts, reliability on the one hand and low material consumption of the latter made with the use of new structural materials. This problem is particularly relevant nowadays and it is of great interest to researchers who solve fundamental and applied problems in the sphere of contact interaction of bodies with initial (residual) stresses. Problems arising with the transfer of stress from the elastic band to regularly placed elastic cover plates – the classical theory of elasticity – are becoming relevant again with the appearance of initial (residual) stress in the band.

Literature review

In spite of the fact that researches of influence of initial tension began to be conducted actively in our country and abroad only at the end of the 20th century, it is possible to list many names, researches and publications of this subject [1, 2]. In stringent formulation of contact problems for elastic bodies with initial stresses, there is a need to involve the apparatus of nonlinear elasticity theory, which greatly complicates the construction of analytical solutions. But in the case of large (finite) stresses (deformations) it is possible to be limited to consideration the linearized theory of elasticity [1]. There are two directions of research of contact problems within the framework of the linearized theory of elasticity. The first is connected with studies of contact interaction of bodies with a concrete form of elastic potential [3]. In the second one - the problem is put in the general form for compressible (noncomputable) bodies with the potential of an arbitrary structure based on the linearized theory of elasticity [1, 2, 6 - 11].

In this article, using the relations of the linearized theory of elasticity [2], the solving of the contact problem for the contact interaction of an infinite and finite stringer with a prestressed strip is presented. The study was carried in a general form for compressible and incompressible bodies for the theory of large (finite) initial strains and two variants of the theory of small initial strains at an arbitrary structure of elastic potential [6, 9].

Aim and Problem Statement

The aim of this work is to study the effect of initial (residual) stresses on the law of distributing stresses and displacement under regularly placed elastic cover plates on the contact line with a pre-stressed band. Let us consider an elastic initially stressed infinite band whose thickness is *t* and which is supported by elastic cover plates h thick regularly placed on its surface plane $y_1 = 0$ at finite intervals $\{-a+2lk,\ a+2lk\}$, $(l > a,\ k = 0 \pm 1, \pm 2 ...)$ with the period of $2l$ while surface plane $y_2 = 0$ is tightly fixed (Fig. 1).

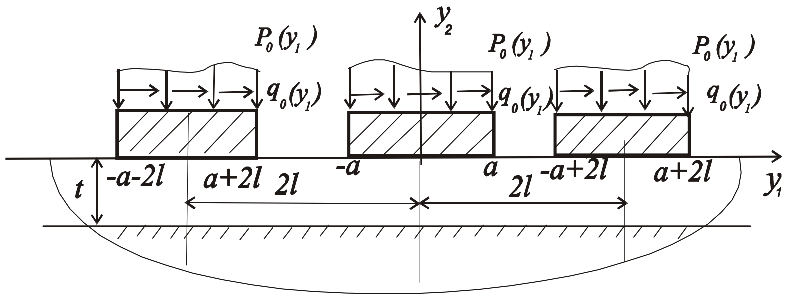

Fig. 1. Band reinforcement

It is necessary to determine the influence of the initial (residual) stresses in the elastic band with initial (residual) stresses on the law of distribution of normal and tangential contact stresses in the contact area of elastic cover plates with an elastic band, when the latter is under effect of vertical and horizontal loads with the intensity $p_0(y_1)$ and $q_0(y_1)$ respectively.

In view of the frequency of the considered problem the influence of initial (residual) stresses on L_k at $y_2 = 0$ under elastic thin cover plates will be equal [5], i.e. contact stresses arising on L_k – intervals from the side of the elastic cover plates will be periodic functions with the period $2l$.

$$\tau_{xy}(y_1)\big|_{y_2=0} = \tau(y_1) = \tau(y_1 - 2l) = \tau(y_1 + 2l);$$
$$\tilde{Q}_{22}(y_1) = Q_{22}(y_1 - 2l) = Q_{22}(y_1 + 2l); \qquad (1)$$
$$Q_{22}\,\tilde{Q}_{21}(y_1) = Q_{21}(y_1 - 2l) = Q_{21}(y_1 + 2l).$$

So, we can confine ourselves to examining one of them, for example, the one for which $k = 0$. We will also consider that there are some valid assumptions for the elastic band with initial (residual) stresses and elastic cover plates, namely: the elastic band with initial (residual) stresses is in the condition of planar deformation, and for the elastic strap loaded both by vertical and horizontal forces the common model of beam bending in combination with the model of a uni-axial stress and strain state of the elastic strap is true. This means that the elastic strap bends like a regular beam in the vertical direction Oy_2, it contracts or stretches in the horizontal direction of the Oy_1 axis like a normal shank in a uni-axial stress and strain state with finite rigidity. From this perspective the discussed problem will have the following equation [4]:

$$u(y_1) = u_1(y_1); \quad v(y_1) = v_2(y_2); \quad y_1 \in (-a + 2kl < y_1 < a + 2kl);$$
$$(k = 0, \pm 1, \pm 2, ...). \qquad (2)$$

Here u, v – are respectively horizontal and vertical movements of the elastic strap in the designations of the classic theory of elasticity; $u_1(y_1)$, $u_2(y_2)$ – are respectively horizontal and vertical movements of the limiting points of the elastic band with initial (residual) stress; $p(y_1)$, $q(y_1)$ – intensity of normal and horizontal contact stress.

Presentation of the main material

1. Deriving singular and integro-differential equations

When deriving the basic system of differential equations that enable us to solve the given problem we should note that based on the principle of superposition the function of influence for vertical and horizontal movement of the limiting points of the elastic band plane $y_2 = 0$ and taking into account the regularity of the latter [5] can be written as:

$$\int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)p(\tau)d\tau=\int_{-a}^{a} h_{11}(|y_1-\tau|)p(\tau)d\tau+\int_{-a-2l}^{a-2l} h_{11}(|y_1-\tau|)p(\tau)d\tau+$$
$$+\int_{-a+2l}^{a+2l} h_{12}(|y_1-\tau|)p(\tau)d\tau+\int_{-a-4l}^{a-4l} h_{11}(|y_1-\tau|)p(\tau)d\tau+\int_{-a+4l}^{a+4l} h_{11}(|y_1-\tau|)p(\tau)d\tau+... \quad (3)$$

For receiving the value of the integral (3) the regularity of normal and tangential contact stresses $p(\tau)$ and $q(\tau)$ is used.

$$\int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)\,p(\tau)d\tau=\int_{-a}^{a} h_{11}(|y_1-\tau|)\,p(\tau)d\tau+\int_{-a-2l}^{a-2l} h_{11}(|y_1-\tau|)\,p(\tau)d\tau+$$
$$+\int_{-a+2l}^{a+2l} h_{12}(|y_1-\tau|)\,p(\tau)d\tau+\int_{-a-4l}^{a-4l} h_{11}(|y_1-\tau|)\,p(\tau)d\tau+$$
$$+\int_{-a+4l}^{a+4l} h_{11}(|y_1-\tau|)\,p(\tau)d\tau+...; \quad (4)$$
$$p(\tau)=p(\tau-2kl)=p(\tau+2kl)=...;$$
$$q(\tau)=q(\tau-2kl)=q(\tau+2kl)=... .$$

Substituting variables $\tau=\eta+2kl,\quad (k=1,\pm1,\pm2,...)$ taking into account the value of displacements under the action of the single normal and tangential force [3] after a series of transformations leads to the following form of the integral (5):

$$\int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p(\tau)d\tau=\frac{1}{2\pi}\int_{-a}^{a}\left\{\int_{-\infty}^{\infty} H_{11}(\alpha)\cos(y_1\eta)s\left[1+2\sum_{k=1}^{\infty}\cos 2kls\right]ds\right\}p(\eta)d\eta. \quad (5)$$

This integral can be some what simplified if you use the value [4]:

$$1+2\sum_{k=1}^{\infty}\cos ky_1=\frac{2\pi}{l}\sum_{k=-\infty}^{\infty}\delta(y-2\pi k);$$
$$\delta(\alpha y_1)=\frac{1}{\alpha}\delta(y_1). \quad (6)$$

Here $\delta(y_1)$ – Dirac unit function [5].

After a series of transformations we obtain:

$$\int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p(\tau)d\tau=\frac{P_0}{2l}H_{11}(0)+\frac{1}{l}\int_{-a}^{a}\sum_{k=1}^{\infty}H_{11}\left(\frac{\pi k}{l}\right)\times\cos\left[\frac{\pi k}{l}(y_1-\eta)\right]p(\eta)d\eta. \quad (7)$$

Applying the same procedure to other integrals we have:

$$\int_{-\infty}^{\infty} h_{12}(|y_1-\tau|)q(\tau)d\tau=\frac{Q_0}{2l}H_{12}(0)+\frac{1}{l}\int_{-1}^{1}\sum_{k=1}^{\infty}H_{12}\left(\frac{\pi k}{l}\right)\times\sin\left[\frac{\pi k}{l}(y_1-\eta)\right]q(\eta)d\eta.$$
$$\int_{-\infty}^{\infty} h_{21}(|y_1-\tau|)p(\tau)d\tau=\frac{P_0}{2l}H_{21}(0)+\frac{1}{l}\int_{-a}^{a}\sum_{k=1}^{\infty}H_{21}\left(\frac{\pi k}{l}\right)\times\sin\left[\frac{\pi k}{l}(y_1-\eta)\right]p(\eta)d\eta. \quad (8)$$
$$\int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)q(\tau)d\tau=\frac{Q_0}{2l}H_{22}(0)+\frac{1}{l}\int_{-a}^{a}\sum_{k=1}^{\infty}H_{22}\left(\frac{\pi k}{l}\right)\times\cos\left[\frac{\pi k}{l}(y_1-\eta)\right]q(\eta)d\eta.$$

Let us write down asymptotic expressions $H_{ij}(\alpha)i$, $j=1,2$, at $\alpha\to0$ i $\alpha\to\infty$.

$$\lim_{\alpha\to0}H_{11}(\alpha)=-A;\quad \lim_{\alpha\to0}H_{12}(\alpha)=O(\alpha);$$
$$\lim_{\alpha\to0}H_{22}(\alpha)=A_1;\quad \lim_{\alpha\to\infty}H_{11}(\alpha)=-B\cdot O(\alpha^{-1}); \quad (9)$$
$$\lim\ \lim_{\alpha\to\infty}H_{22}(\alpha)=B\cdot O(\alpha^{-1});\quad \lim_{\alpha\to\infty}H_{12}(\alpha)=-B_1\cdot O(\alpha^{-1}).$$

Here A, A_1, B, B_2 are values characterizing the initial (residual) stress and strain state in the elastic band and are determined for compressible and noncompressible bodies in the case of a specific structure of elastic potentials.

If we consider the value of the sum given below [4]:

$$\sum_{k=1}^{\infty} \frac{\cos kx}{k} = -\ln\left|2\sin\frac{x}{2}\right|, \tag{10}$$

then integrals (5) and (8) can be expressed in the following form

$$\begin{aligned} &\int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p(\tau)d\tau = \frac{A}{2l}P_0 + \frac{B}{\pi}\int_{-a}^{a} p(\tau)\ln\left|2\sin\frac{\pi}{l}\left(\frac{y_1-\eta}{2}\right)\right|dt + \\ &+\frac{1}{l}\int_{-a}^{a}\sum_{k=1}^{\infty}\left\{\left[H_{11}\left(\frac{\pi k}{l}\right)+\frac{Bl}{\pi k}\right]\cos\frac{\pi k}{l}(y_1-\eta)\right\}p(\eta)d\eta; \\ &\int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)q(\tau)d\tau = -\frac{A_1}{2l}Q_0 - \frac{B}{\pi}\int_{-a}^{a} q(\tau)\ln\left|2\sin\frac{\pi}{l}\left(\frac{y_1-\eta}{2}\right)\right|d\eta + \\ &+\frac{1}{l}\int_{-a}^{a}\sum_{k=1}^{\infty}\left\{\left[H_{22}\left(\frac{\pi k}{l}\right)+\frac{Bl}{\pi k}\right]\cos\frac{\pi k}{l}(y_1-\eta)\right\}q(\eta)d\eta. \end{aligned} \tag{11}$$

$$\sum_{k=1}^{\infty}\frac{\sin ky_1}{k} = \frac{\pi-|y_1|}{2}\sin qy_1 \quad (-2\pi < y < 2\pi), \tag{12}$$

integrals (11) can be written as the formulas below:

$$\begin{aligned} &\int_{-\infty}^{\infty} h_{12}(|y_1-\tau|)q(\tau)d\tau = \frac{1}{l}\int_{-a}^{a}\left\{\sum_{k=1}^{\infty}\left[H_{12}\left(\frac{\pi k}{l}\right)+\frac{B_1 l}{\pi k}\right]\sin\left[\sin\frac{\pi k}{l}(y_1-\eta)\right]\right\}q(\eta)d\eta - \\ &-\frac{B_1}{2l}\int_{-a}^{a} q(\tau)\sin q(y_1-\eta)d\eta + \frac{B_1}{2l}\left[Q_0 y_1 - \overline{M}_0\right]; \quad |y_1| \le a, \\ &\int_{-\infty}^{\infty} h_{21}(|y_1-\tau|)p(\tau)d\tau = \frac{1}{l}\int_{-a}^{a}\left\{\sum_{k=1}^{\infty}\left[H_{21}\left(\frac{\pi k}{l}\right)+\frac{B_1 l}{\pi k}\right]\sin\left[\frac{\pi k}{l}(y_1-\eta)\right]\right\}p(\eta)d\eta - \\ &-\frac{B_1}{2l}\int_{-a}^{a} p(\tau)\sin q(y_1-\eta)d\eta + \frac{B_1}{2l}\left[P_0 y_1 - M_0\right]; \quad |y_1| \le a. \end{aligned} \tag{13}$$

where

$$\overline{M}_0 = \int_{-a}^{a} x\,q(x)dx, \quad M_0 = \int_{-a}^{a} x\,p(x)d\xi. \tag{14}$$

Taking into account (8), (11), (13) we find the displacement of the limiting points in the contact area $y_1 \in [-a, a]$ on the surface plane $y_2 = 0$ of the elastic band with initial (residual) stress.

$$\begin{aligned} &u_1(y_1) = -\frac{B}{\pi}\int_{-a}^{a} q(\eta)\ln\left[2\sin\frac{\pi}{2l}(y_1-\eta)\right]d\eta - \frac{B_1}{2} + \\ &+\int_{-a}^{a} L_{12}(|y_1-t|)p(\eta)d\eta + \int_{-a}^{a} L_{12}(|y_1-t|)q(\eta)d\eta - \frac{1}{2l}\left[B_1\left(Q_0 y - \overline{M}_0\right) - AP_0\right], \\ &\qquad |y_1| < a. \\ &u_2(y_1) = -\frac{B}{\pi}\int_{-a}^{a} q(\eta)\ln\left[2\sin\frac{\pi}{2l}(y_1-\eta)\right]d\eta - \frac{B_1}{2}\int_{-a}^{a} p(\eta)\,sign(y_1-\eta)d\eta + \\ &+\int_{-a}^{a} L_{22}(|y_1-\eta|)q(\eta)d\eta + \int_{-a}^{a} N_{12}(|y_1-\eta|)p(\eta)d\eta - \frac{1}{2l}\left[B_1(P_0 y - M_0) - A_1 Q_0\right]. \end{aligned} \tag{15}$$

We introduce a designation here:

$$L_{11}\left(\left|y_1-\eta\right|\right)=\frac{1}{l}\sum_{k=1}^{\infty}\left[H_{11}\left(\frac{\pi k}{l}\right)+\frac{Bl}{\pi k}\right]\cos\frac{\pi k}{l}\left(y_1-\eta\right);$$
$$L_{12}\left(y_1-\eta\right)=\frac{1}{l}\sum_{k=1}^{\infty}\left[H_{12}\left(\frac{\pi k}{l}\right)+\frac{B_1 l}{\pi k}\right]\sin\frac{\pi k}{l}\left(y_1-\eta\right); \tag{16}$$
$$L_{22}\left(\left|y_1-\eta\right|\right)=\frac{1}{l}\sum_{k=1}^{\infty}\left[H_{22}\left(\frac{\pi k}{l}\right)+\frac{Bl}{\pi k}\right]\cos\frac{\pi k}{l}\left(y_1-\eta\right).$$

In order to obtain initial functional equations, from which unknown contact stresses are determined (taking into account (15)) we have to make the following system of singular integro-differential equations.

$$D\frac{d^4}{dy_1^4}\left\{\frac{B}{\pi}\int_{-a}^{a}p(\eta)\ln\left|2\sin\frac{\pi}{l}\left(\frac{y_1-\eta}{2}\right)\right|d\eta-\frac{B_1}{2}\int_{-a}^{a}q(\eta)sign\left(y_1-\eta\right)d\eta+\right.$$
$$\left.+\int_{-a}^{a}L_{11}\left(\left|y_1-\eta\right|\right)p(\eta)d\eta+\int_{-a}^{a}L_{12}\left(\left|y_1-\eta\right|\right)q(\eta)d\eta+\frac{1}{2l}\left[B_1\left(Q_0 y-\overline{M}_0\right)-AP_0\right]\right\}=p\left(y_1\right)-p_0\left(y_1\right).$$
$$\frac{d}{dy_1}=\left\{\frac{B}{\pi}\int_{-a}^{a}q(\eta)d\eta\ln\left[2\sin\frac{\pi}{l}\left(\frac{y_1-\eta}{2}\right)\right]d\eta-\frac{B_1}{2}\int_{-a}^{a}p(\eta)sign\left(y_1-\eta\right)d\eta+\right.$$
$$\left.+\int_{-a}^{a}L_{22}\left(\left|y_1-\eta\right|\right)q(\eta)d\eta+\int_{-a}^{a}L_{12}\left(\left|y_1-\eta\right|\right)p(\eta)d\eta+\frac{1}{2l}\left[B_0\left(P_0 y-M_0\right)-A_1P_0\right]\right\}=$$
$$=\frac{1}{E_1 h}\int_{-a}^{a}\left[q(\eta)-q_0(\eta)\right]d\eta,\ \ \left|y_1\right|\le a. \tag{17}$$

This system is solved with such boundary conditions:
where $y_1=a$

$$\int_{-a}^{a}\eta p(\eta)d\eta=M_0;\ \ \int_{-a}^{a}q(\eta)d\eta=Q_0;\ \ \int_{-a}^{a}p(\eta)d\eta=P_0. \tag{18}$$

2. Bringing a singular integro-differential equation to an infinite system of linear algebraic equations
The system of singular integral equations can be simplified by introducing new functions.

$$X(\tau)=\tilde{p}(\tau)+i\tilde{q}(\tau);$$
$$\tilde{p}(\tau)=p\left(\frac{l\tau}{\pi}\right)\frac{1}{P_0};\ \ \tilde{q}(\tau)=q\left(\frac{l\tau}{\pi}\right)\frac{1}{Q_0}; \tag{19}$$
$$\tilde{p}_0(\tau)=p\left(\frac{l\tau}{\pi}\right)\frac{1}{P_0};\ \ \tilde{q}_0(\tau)=q_0\left(\frac{l\tau}{\pi}\right)\frac{1}{Q_0}.$$

After complex cumbersome transformations and replacement of variables

$$\frac{\pi}{l}y_1=\tau,\ \ \frac{\pi}{l}\eta=\xi \tag{20}$$

we obtain an infinite system of linear algebraic equations:

$$l_m\tilde{X}_n+\sum_{n=1}^{\infty}\left[P_{m,n}^{(1)}\tilde{X}_n+P_{m,n}^{(2)}\overline{X}_n\right]=-\left[P_m^{(1)}\tilde{X}_0+P_m^{(2)}\overline{X}_0+P_m^{(3)}+R\cdot P_m^4\right],\ (m=1,2,...) \tag{21}$$

For using numerical methods to the system (21) it is necessary to determine the coefficient $\overline{X}_0$ and constant R, which are in the right part.

$$\overline{X}_0=\frac{ch\,\pi\alpha_1\cdot\cos\frac{\delta}{2}}{2l}\left(1+i\frac{Q_0}{P_0}\right). \tag{22}$$

Constant R can be found from a system of algebraic equations (21) which is completely quasiregular.

Conclusions

For obtaining a numerical result we have studied a case where all regularly placed cover plates that reinforce the elastic band with initial (residual) stresses are loaded with tangential force $Q_0(\alpha) = Q$.

Fig. 2, 3, 4 illustrate the influence of initial (residual) stresses in the elastic band on the law of distributing contact stresses under regularly placed cover plates from the action of tangential force $q_0(y_1) = Q_0$ for dimentionless values (19). The graphs show the place of curves for the parameter value $\lambda_1 = 1,3; 1,2; 1,1; 1; 0,9; 0,8; 0,7$ which characterizes the initial (residual) stress and deformed state. Value $\lambda_1 = 1$ corresponds to an elastic band without initial (residual) stress and on the graphs it is depicted as a dotted line, i.e., the value of contact stresses for a similar problem solved within the classical theory of elasticity.

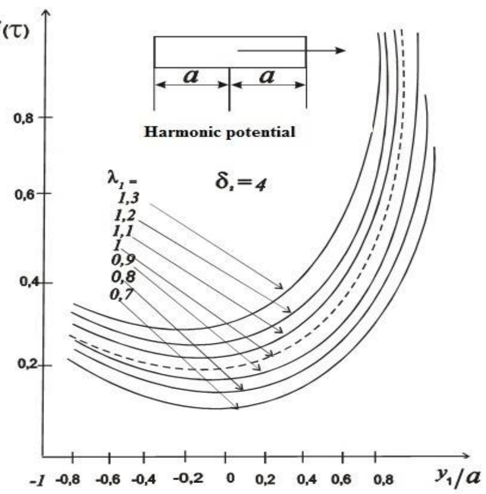

Fig. 2. Distribution of loads under the finite stringers (Harmonic potential)

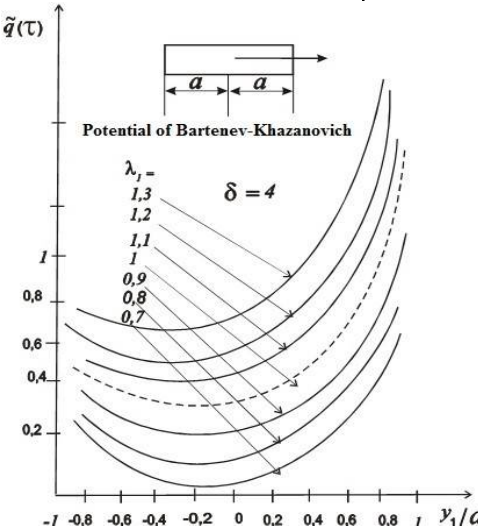

Fig. 3. Distribution of loads under the finite stringers (Potential of Bartenev-Khazanovich)

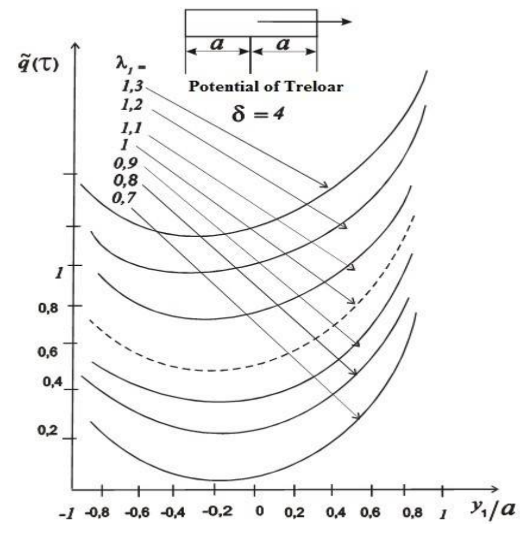

Fig. 4 . Distribution of loads under the finite stringers (Potential of Treloar)

It follows from the graphs that the regularity of the influence of initial stresses in the elastic band on the distribution of contact tangential stresses essentially depends on the structure of the elastic potential in the latter. Initial stresses have a more significant quantitative effect in highly elastic materials compared with rigid ones, the qualitative effects subside. Moreover, at compression $(\lambda_1 < 1)$ contact stresses are reduced, and in the case of stretching $(\lambda_1 > 1)$ they increase. This result can be efficiently used to regulate contact forces when calculating durability for designing constructions.

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Діхтярук М.М., Курінєнко О.В., Поплавська О.А. Контактна задача про взаємодію смуги з початковими напруженнями з періодично розміщеними пружними накладками

В даній статті розглядається постановка і розв'язок задачі про контактну взаємодію попередньо напруженої смуги з періодично розміщеними пружними накладками. Всі дослідження проводилося в рамках лінеаризованої теорії пружності у загальному вигляді для теорії великих (скінчених) початкових деформацій та двох варіантів теорії малих початкових деформацій при довільній структурі пружного потенціалу [1]. Розв'язок задачі зводиться до розв'язування сингулярного інтегро-диференційного рівняння для невідомих контактних напружень з ядром, записаним у вигляді суми ядра і деякого регулярного ядра, при певних граничних умовах. Шуканий розв'язок подано у вигляді ряду за поліномами Якобі і для визначення невідомих коефіцієнтів ряду отримана цілком квазірегулярна нескінчена система лінійних рівнянь [2].

Ключові слова: лінеаризована теорія пружності, початкові (залишкові) напруження, контактні задачі, інтегральні перетворення Фур'є.