UDC 519.832.3

V. V. Romanuke (cand. of tech. sc.)

Khmelnytskyy National University
romanukevadimv@mail.ru

A PROGRAM TOOL FOR PRACTICING THE MODEL OF SEARCHING AN ACTIVE OBJECT AS MULTISTAGE DIAGONAL GAME

There has been represented a model of searching an active object within a closed checkered area as multistage diagonal game. The evolving from stage to stage solution of this game may be found with the developed and represented program tool. The tool returns the solutions and plots bar-like distribution of their elements up with the prior sojourn probabilities distribution over the area.

searching an object, diagonal game, multistage optimization, optimal behavior, Matlab program tool

Introduction and problem statement

Many domains of activity have the arisen problems of searching something or somebody within a closed territorial region. One of the most outstanding and really needful is the state frontier problems of searching trespassers. Another one is of searching the lost people, especially kids. This is why to model the search of an active object, moving or staying lost within some territorial region, turns to be the actual scientific problem, which has the real practice use.

Analysis of investigations on territorial region search models

Amazingly enough, but territorial region search models had not been represented much until now. It might be explained with that when somebody is getting lost or trespasses the state frontier, then there is no much time to evaluate the environmental situation, and it is very important to act as fast as possible. However, as for the state frontiers environmental situations, they are almost stable in their maneuverability through. And so, there is possibility to premodel the trespassing process through them, what will help to react faster in extraordinary situations [1, 2]. This also may concern the model of the enemy troops invasion in trespassing the state frontier. Besides, some types of illegal migration may be modeled in that way [3]. But still the unsolved part of the problem is in developing the program tool for practicing the model of the active object search, and that model must regard the worst situations as the object may not want to be exposed (except cases of searching the lost people or kids, though there are those worst situations should be considered anyway).

Paper purpose and tasks formulation

Let the trespasser or the lost one, for further being called the object, moves across the rectangular region (area) Z. This region should be divided into M lines, each of which is divided into N zones (squares). For i-th line there is known the prior probability p_{ij} of the object sojourn within the j-th zone, that is

$$\sum_{j=1}^{N} p_{ij} = 1 \quad \forall\, i = \overline{1, M}. \tag{1}$$

Here it should be stated, that on the first line there is the condition

$$p_{1j} \geqslant 0 \quad \forall\, j = \overline{1, N},\ \exists\, j_1 \in \left\{\overline{1, N}\right\},\ \exists\, j_2 \in \left\{\overline{1, N}\right\},\ j_1 \neq j_2,\ p_{1j_1} > 0,\ p_{1j_2} > 0. \tag{2}$$

It is because of that if (2) is false, then on the first line there exists a zone $j_1 \in \left\{\overline{1, N}\right\}$ with the unit probability $p_{1j_1} = 1$, and the problem is already solved: it remains to just direct some search group to locate it there.

The object, entering or trespassing the area Z through the first line in a zone of those N ones, knows the prior probabilities $\left\{p_{1j}\right\}_{j=1}^{N}$ as they are learned by the search group. Then the object will act most cautiously, taking into consideration the possible best searching actions against it. The search group will act analogously, minding the best camouflaging or prudent actions of the trespasser. Hence this event is antagonistic, being modeled on the first line as matrix $N \times N$-game, where the k-th pure strategy x_k of the search group (first player) is in directing to the k-th zone, and l-th pure strategy y_l of the trespasser (second player) is in moving to the l-th zone, by $k = \overline{1, N}$ and $l = \overline{1, N}$. Such game is diagonal, as in the situation $\left\{x_k, y_l\right\}$ the search group payoff is zeroth by $k \neq l$ and is p_{1k} by $k = l$. This game kernel is the matrix $\mathbf{D}(1) = \left[d_{kl}(1)\right]_{N \times N}$ with elements

$$d_{kl}(1) = p_{1k}\left(1 - \operatorname{sign}\left|k - l\right|\right) \text{ by } k = \overline{1, N} \text{ and } l = \overline{1, N}. \tag{3}$$

Knowing the diagonal $\mathbf{D}(1)$-game solution [4]

$$\mathbf{S}_{\text{opt}}(1)=\left[s_{\text{opt}}^{\langle 1\rangle}(1)\quad s_{\text{opt}}^{\langle 2\rangle}(1)\quad \dots\quad s_{\text{opt}}^{\langle N-1\rangle}(1)\quad s_{\text{opt}}^{\langle N\rangle}(1)\right]\in$$
$$\in\left\{\left[s^{\langle 1\rangle}(1)\quad s^{\langle 2\rangle}(1)\quad \dots\quad s^{\langle N-1\rangle}(1)\quad s^{\langle N\rangle}(1)\right]\in\mathbb{R}^N:\right.$$
$$\left. s^{\langle k\rangle}(1)\in[0;\,1]\;\forall\,k=\overline{1,\,N},\;\sum_{k=1}^{N}s^{\langle k\rangle}(1)=1\right\} \tag{4}$$

on the first line, where for the search group $s_{\text{opt}}^{\langle k\rangle}(1)$ is the optimal probability of searching the trespasser in the k-th zone, and for the trespasser $s_{\text{opt}}^{\langle l\rangle}(1)$ is the optimal probability of sojourning within the l-th zone, and

$$s_{\text{opt}}^{\langle j\rangle}(1)=\frac{\left[d_{jj}(1)\right]^{-1}}{\sum_{k=1}^{N}\left[d_{kk}(1)\right]^{-1}}=\frac{\left(p_{1j}\right)^{-1}}{\sum_{k=1}^{N}\left(p_{1k}\right)^{-1}}\quad\forall\,j=\overline{1,\,N}, \tag{5}$$

there is the task to develop a program tool for practicing the optimal behavior model of searching the active object on deeper lines. By that the optimal behavior principle should be based on the multistage diagonal game solution, being regenerated on each new line within the area Z.

Regenerated diagonal game and optimal behavior principle

To behave optimally on the first line means to hold the mixed strategy (4) for both the search group and object. Generally, this implies to search with probability $s_{\text{opt}}^{\langle k\rangle}(1)$ in the k-th zone. But, actually, this may be comprehended as to divide the search group into parts with proportional to probabilities $\left\{s_{\text{opt}}^{\langle j\rangle}(1)\right\}_{j=1}^{N}$ fractions. For instance, if $\mathbf{S}_{\text{opt}}(1)=\left[\frac{1}{7}\quad\frac{4}{7}\quad\frac{2}{7}\right]$ then, having the search group 21 members, there are directed 3 persons into the first zone, 12 persons into the second, and 6 persons into the third zone of the first line.

If it occurred that the object trespassed the first line, then its optimal behavior model must not be changed: there will be the same best camouflaging or prudent actions. Then the search group should act similarly, taking into consideration the most cautious movements of the object. And here the diagonal

$N\times N$-game regenerates, but not only regarding the probabilities $\left\{p_{2j}\right\}_{j=1}^{N}$. These probabilities naturally must be weighted with the probabilities $\left\{s_{\text{opt}}^{\langle j\rangle}(1)\right\}_{j=1}^{N}$. The second line game kernel is the matrix $\mathbf{D}(2)=\left[d_{kl}(2)\right]_{N\times N}$ with elements

$$d_{kl}(2)=s_{\text{opt}}^{\langle k\rangle}(1)\,p_{2k}\left(1-\operatorname{sign}|k-l|\right) \text{ by } k=\overline{1,N} \text{ and } l=\overline{1,N} \tag{6}$$

with its solution

$$\mathbf{S}_{\text{opt}}(2)=\left[s_{\text{opt}}^{\langle 1\rangle}(2)\quad s_{\text{opt}}^{\langle 2\rangle}(2)\quad \ldots\quad s_{\text{opt}}^{\langle N-1\rangle}(2)\quad s_{\text{opt}}^{\langle N\rangle}(2)\right]\in$$
$$\in\left\{\left[s^{\langle 1\rangle}(2)\quad s^{\langle 2\rangle}(2)\quad \ldots\quad s^{\langle N-1\rangle}(2)\quad s^{\langle N\rangle}(2)\right]\in\mathbb{R}^{N}:\right.$$
$$\left. s^{\langle k\rangle}(2)\in[0;1]\ \forall\ k=\overline{1,N},\ \sum_{k=1}^{N}s^{\langle k\rangle}(2)=1\right\}, \tag{7}$$

$$s_{\text{opt}}^{\langle j\rangle}(2)=\frac{\left[d_{jj}(2)\right]^{-1}}{\sum_{k=1}^{N}\left[d_{kk}(2)\right]^{-1}}=\frac{\left[s_{\text{opt}}^{\langle j\rangle}(1)\,p_{2j}\right]^{-1}}{\sum_{k=1}^{N}\left[s_{\text{opt}}^{\langle k\rangle}(1)\,p_{2k}\right]^{-1}}\quad \forall\ j=\overline{1,N}. \tag{8}$$

Note, that elements (6) of $\mathbf{D}(2)$-game matrix, positioned on the main diagonal, are not probabilities now. Properly speaking, the probability of the object sojourn within the j-th zone $\forall\ j=\overline{1,N}$ of the second line is

$$\frac{s_{\text{opt}}^{\langle j\rangle}(1)\,p_{2j}}{\sum_{l=1}^{N}s_{\text{opt}}^{\langle l\rangle}(1)\,p_{2l}}=\frac{\dfrac{\left(p_{1j}\right)^{-1}}{\sum_{k=1}^{N}\left(p_{1k}\right)^{-1}}\,p_{2j}}{\sum_{l=1}^{N}\dfrac{\left(p_{1l}\right)^{-1}}{\sum_{k=1}^{N}\left(p_{1k}\right)^{-1}}\,p_{2l}}=\frac{\dfrac{p_{2j}}{p_{1j}}}{\sum_{l=1}^{N}\dfrac{p_{2l}}{p_{1l}}}, \tag{9}$$

but it is not necessary to recalculate it, as the corresponding diagonal game solution will be the same, though the game optimal value, being here out of interest, is changed.

Generally, if it occurred that the object trespassed the i-th line $\forall\, i=\overline{1, M-1}$, then there regenerates the diagonal $N\times N$-game with the kernel $\mathbf{D}(i+1)=\left[d_{kl}(i+1)\right]_{N\times N}$ elements

$$d_{kl}(i+1)=s_{\text{opt}}^{\langle k\rangle}(i)\,p_{(i+1)k}\left(1-\operatorname{sign}|k-l|\right) \text{ by } k=\overline{1, N} \text{ and } l=\overline{1, N} \tag{10}$$

by its solution

$$\mathbf{S}_{\text{opt}}(i+1)=\left[s_{\text{opt}}^{\langle 1\rangle}(i+1)\;\; s_{\text{opt}}^{\langle 2\rangle}(i+1)\;\; \ldots\;\; s_{\text{opt}}^{\langle N-1\rangle}(i+1)\;\; s_{\text{opt}}^{\langle N\rangle}(i+1)\right]\in$$

$$\in\left\{\left[s^{\langle 1\rangle}(i+1)\;\; s^{\langle 2\rangle}(i+1)\;\; \ldots\;\; s^{\langle N-1\rangle}(i+1)\;\; s^{\langle N\rangle}(i+1)\right]\in\mathbb{R}^{N}:\right.$$

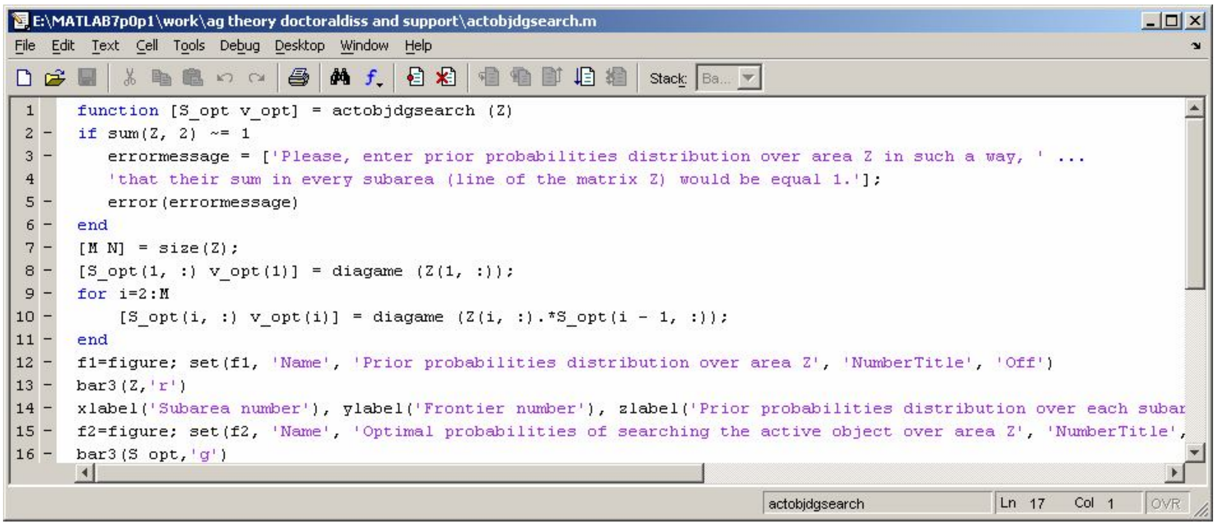
$$\left. s^{\langle k\rangle}(i+1)\in[0;\,1]\;\forall\, k=\overline{1, N},\; \sum_{k=1}^{N} s^{\langle k\rangle}(i+1)=1\right\}, \tag{11}$$

$$s_{\text{opt}}^{\langle j\rangle}(i+1)=\frac{\left[d_{jj}(i+1)\right]^{-1}}{\sum\limits_{k=1}^{N}\left[d_{kk}(i+1)\right]^{-1}}=\frac{\left[s_{\text{opt}}^{\langle j\rangle}(i)\,p_{(i+1)j}\right]^{-1}}{\sum\limits_{k=1}^{N}\left[s_{\text{opt}}^{\langle k\rangle}(i)\,p_{(i+1)k}\right]^{-1}}\;\;\forall\, j=\overline{1, N}. \tag{12}$$

For the search group $s_{\text{opt}}^{\langle k\rangle}(i+1)$ is the optimal probability of searching the object in the k-th zone, and for the object $s_{\text{opt}}^{\langle l\rangle}(i+1)$ is the optimal probability of sojourning within the l-th zone of the $(i+1)$-th line of the area Z. By this the search group may be divided proportionally to probabilities $\left\{s_{\text{opt}}^{\langle j\rangle}(i+1)\right\}_{j=1}^{N}$ as holding the optimal behavior principle in (11).

Program tool for obtaining the optimal behavior guidance

It is very comfortable to use the powerful programmable environment Matlab for technical computations [5, 6]. There is the developed Matlab program tool "actobjdgsearch" for solving the multistage diagonal $\left\{\mathbf{D}(i)\right\}_{i=1}^{M}$-game with diagonals (3) and (10) $\forall\, i=\overline{1, M-1}$. Its code is on figure 1.



E:\MATLAB7p0p1\work\ag theory doctoraldiss and support\actobjdgsearch.m

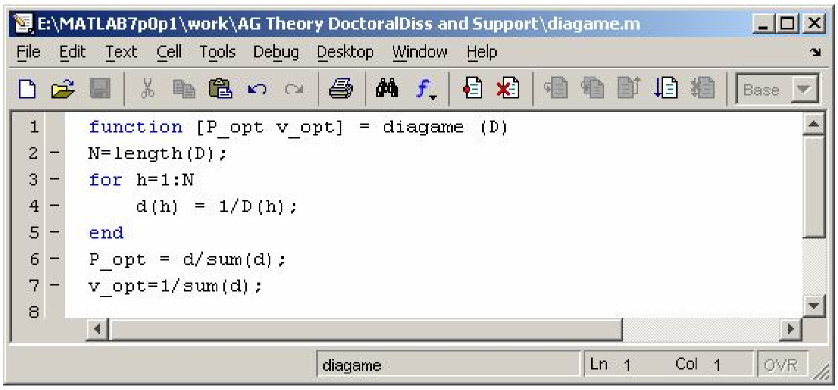
File Edit Text Cell Tools Debug Desktop Window Help

```
1    function [S_opt v_opt] = actobjdgsearch (Z)
2 -  if sum(Z, 2) ~= 1
3 -     errormessage = ['Please, enter prior probabilities distribution over area Z in such a way, ' ...
4       'that their sum in every subarea (line of the matrix Z) would be equal 1.'];
5 -     error(errormessage)
6 -  end
7 -  [M N] = size(Z);
8 -  [S_opt(1, :) v_opt(1)] = diagame (Z(1, :));
9 -  for i=2:M
10 -     [S_opt(i, :) v_opt(i)] = diagame (Z(i, :).*S_opt(i - 1, :));
11 - end
12 - f1=figure; set(f1, 'Name', 'Prior probabilities distribution over area Z', 'NumberTitle', 'Off')
13 - bar3(Z,'r')
14 - xlabel('Subarea number'), ylabel('Frontier number'), zlabel('Prior probabilities distribution over each subar
15 - f2=figure; set(f2, 'Name', 'Optimal probabilities of searching the active object over area Z', 'NumberTitle',
16 - bar3(S_opt,'g')
```

actobjdgsearch Ln 17 Col 1 OVR

Figure 1 — Matlab program tool “actobjdgsearch” code

This tool needs the input prior probabilities matrix $\left\{\left\{p_{ij}\right\}_{j=1}^{N}\right\}_{i=1}^{M}$ to be typed before running it. And it returns for the user the set of solutions $\left\{\mathbf{S}_{\text{opt}}(i)\right\}_{i=1}^{M}$ and, if needed so, the game optimal value on each line (though, repeatedly, it does not any matter here). Also there are returned graphically the prior probabilities $\left\{\left\{p_{ij}\right\}_{j=1}^{N}\right\}_{i=1}^{M}$ distribution over area Z and the optimal probabilities $\left\{\left\{s_{\text{opt}}^{\langle j\rangle}(i)\right\}_{j=1}^{N}\right\}_{i=1}^{M}$ distribution of searching the active object over this area (lines 12 — 17 of the code). Essentially, the tool “actobjdgsearch” forms the kernels $\left\{\mathbf{D}(i)\right\}_{i=1}^{M}$ of the multistage diagonal $N \times N$-game, which is solved within the Matlab program subtool “diagame” (lines 8 and 10 of the code). This diagonal $N \times N$-game solver code is pretty simple (figure 2), calculating just (5) and (12) $\forall\, i = \overline{1, M-1}$.



E:\MATLAB7p0p1\work\AG Theory DoctoralDiss and Support\diagame.m

File Edit Text Cell Tools Debug Desktop Window Help

```
1    function [P_opt v_opt] = diagame (D)
2 -  N=length(D);
3 -  for h=1:N
4 -     d(h) = 1/D(h);
5 -  end
6 -  P_opt = d/sum(d);
7 -  v_opt=1/sum(d);
8
```

diagame Ln 1 Col 1 OVR

Figure 2 — Matlab program subtool “diagame” code

Instancing the developed tool application, consider the area Z with four line frontiers, broken into six zones in each. Let the prior probabilities $\left\{ \left\{ p_{ij} \right\}_{j=1}^{6} \right\}_{i=1}^{4}$ distribution be the following:

$$\left\{ p_{1j} \right\}_{j=1}^{6} = \left\{ \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right\},$$

$$\left\{ p_{2j} \right\}_{j=1}^{6} = \left\{ 0.1, 0.2, 0.2, 0.2, 0.2, 0.1 \right\},$$

$$\left\{ p_{3j} \right\}_{j=1}^{6} = \left\{ 0.1, 0.1, 0.4, 0.1, 0.2, 0.1 \right\},$$

$$\left\{ p_{4j} \right\}_{j=1}^{6} = \left\{ 0.1, 0.1, 0.1, 0.2, 0.3, 0.2 \right\}. \quad (13)$$

One may note the equiprobability within the first line frontier, that might correspond to real conditions. After having typed (13) in the Matlab Command Window prompt line (figure 3), run the tool “actobjdgsearch” and get the results (figures 4, 5 and 6).

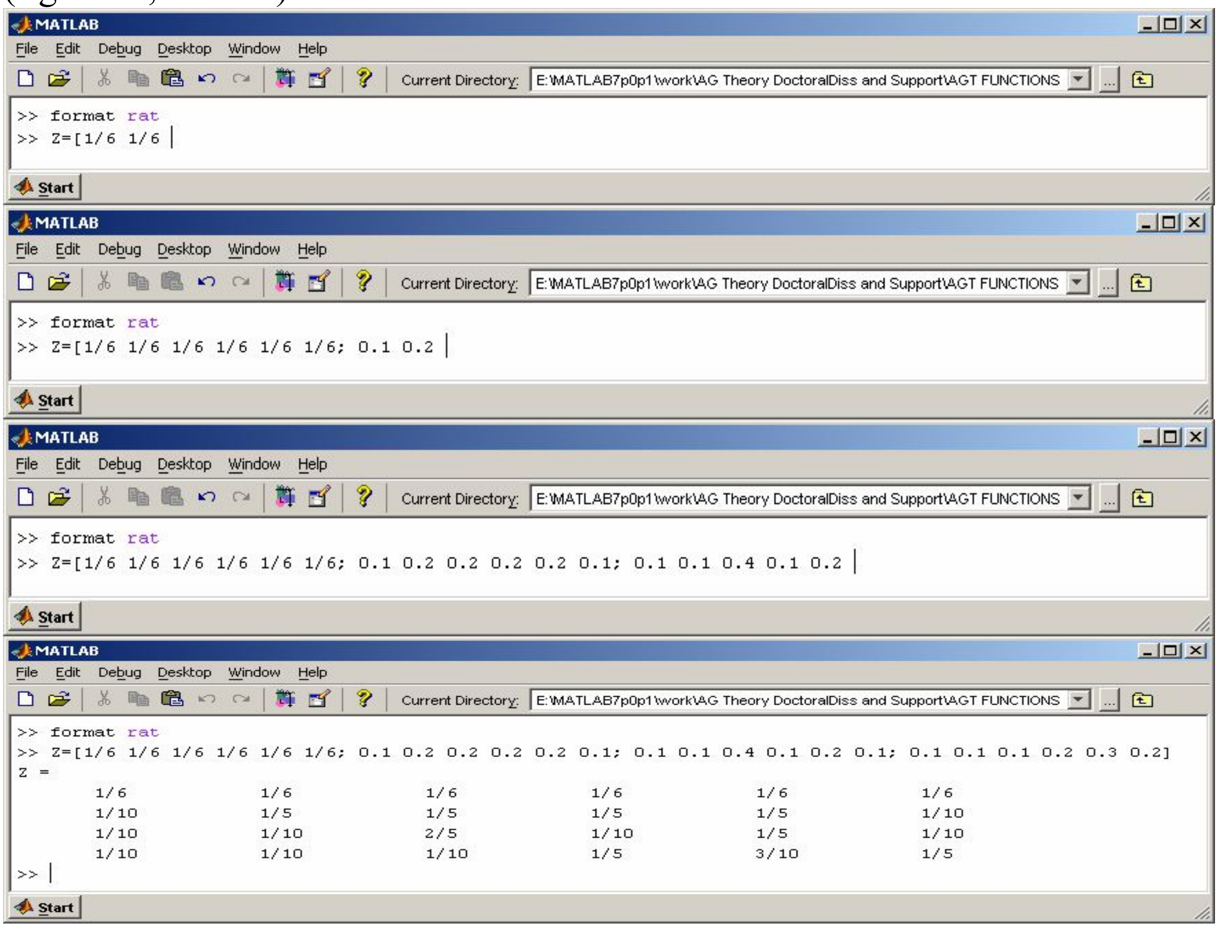


Figure 3 — Typing the prior probabilities (13) in Matlab Command Window prompt line before running “actobjdgsearch”

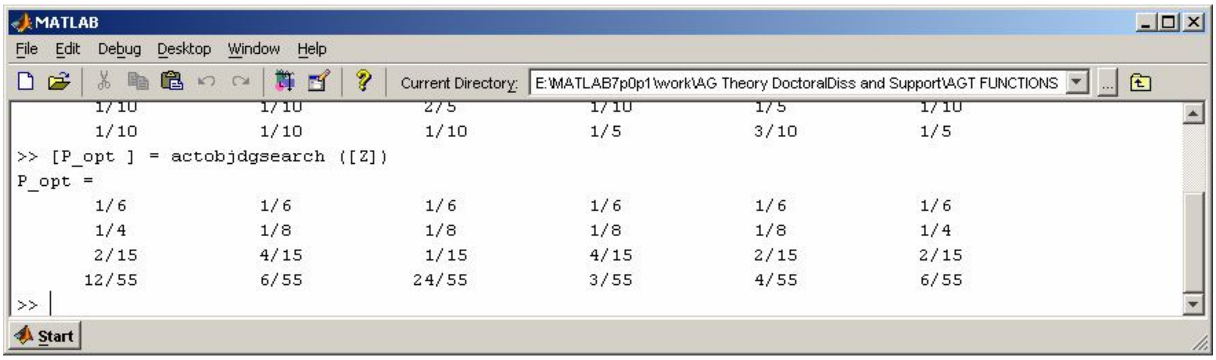


Figure 4 — The set of solutions $\left\{\mathbf{S}_{\text{opt}}(i)\right\}_{i=1}^{4}$ to the prior probabilities (13)

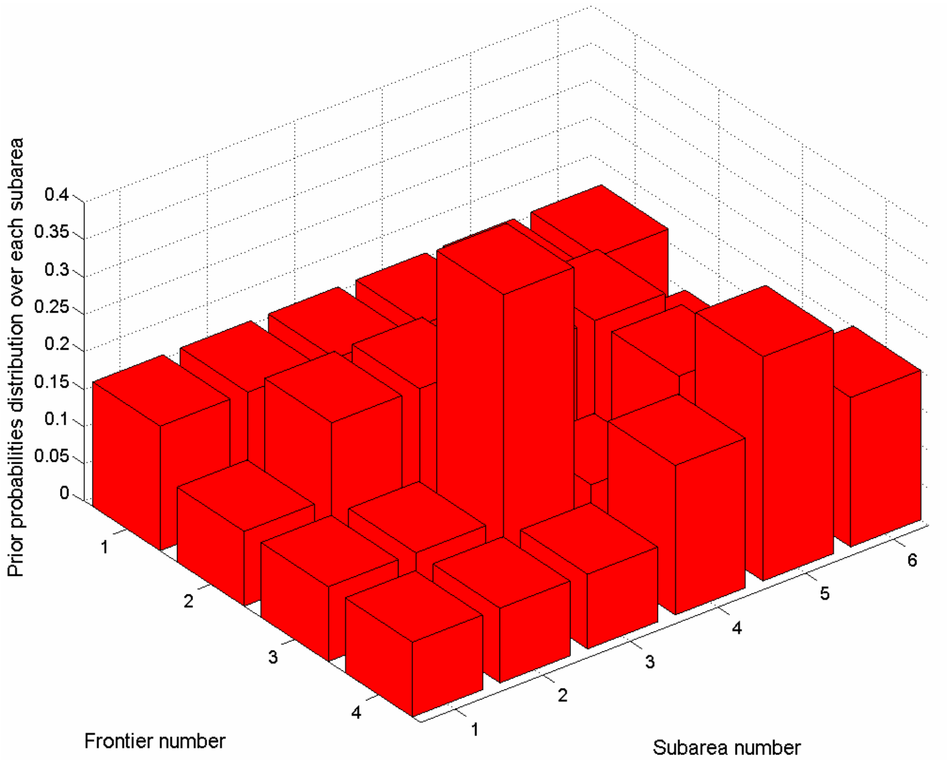


Figure 5 — Graphical return of the prior probabilities (13) distribution over area Z

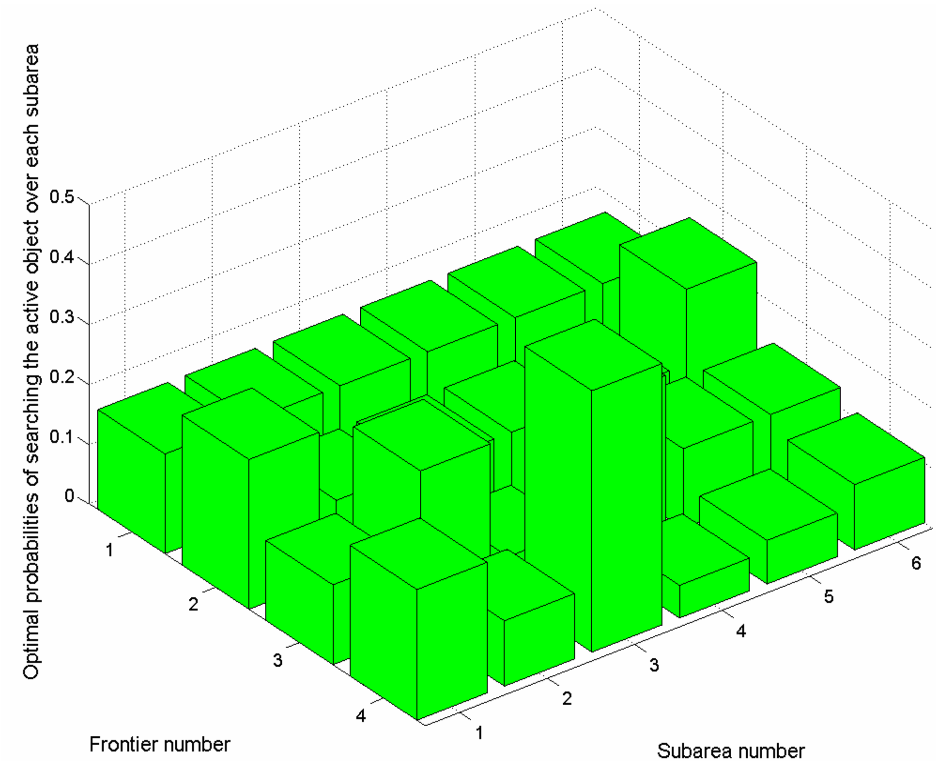


Figure 6 — Graphical return of the optimal probabilities $\left\{\left\{s_{\mathrm{opt}}^{\langle j\rangle}(i)\right\}_{j=1}^{6}\right\}_{i=1}^{4}$ distribution of searching the active object over area Z

It is remarkable here, that the solution $\left\{\mathbf{S}_{\mathrm{opt}}(1)\right\}_{i=1}^{4}$ does not differ from the prior probabilities $\left\{p_{1j}\right\}_{j=1}^{6}$ as they are equal. That is on the first frontier line the distribution of search resource should be equal. Farther the irregularity begins as the territorial region may be of forests, houses, check-points (here, obviously, a prior probability must be low), fields and some. By that the search resource distribution is realized [7, 8] the best of all, if the first line squad staff is of $6Q$ persons, the second line squad staff is of $8Q$ persons, the third line squad staff is of $15Q$ persons, and the fourth line squad staff is of $55Q$ persons by $Q \in \mathbb{N}$.

Conclusions and outlooks of further investigations

There has been modeled the sojourn and moving through the area Z broken into M lines, each of which is divided into N zones, of an active object, could be called a trespasser of the state frontier or just a lost one, which is searched by the qualified squad staff. The suggested model is the diagonal $N \times N$-game, having different kernel from the line to line, until it is the last. The evolving kernel of this game is formed from the prior probabilities $\left\{ \left\{ p_{ij} \right\}_{j=1}^{N} \right\}_{i=1}^{M}$ of the object sojourn within Z. To solve that multistage diagonal game there has been applied the powerful programmable environment Matlab with the developed tool “actobjdgsearch”. On the first frontier line the optimal behavior of both players is in holding at the optimal strategy (4) with (5) as the diagonal $\mathbf{D}(1)$-game solution. On the i-th frontier line $\forall\, i = \overline{1, M-1}$ the optimal behavior of both players is in holding at the optimal strategy (11) with (12) as the diagonal $\mathbf{D}(i+1)$-game solution. The developed and represented Matlab tool “actobjdgsearch” returns those solutions and plots bar-like distributions of them alongside of the prior probabilities $\left\{ \left\{ p_{ij} \right\}_{j=1}^{N} \right\}_{i=1}^{M}$ distribution over Z. The further investigations outlooks are at developing tools for practicing the model of searching an active object with periodically incoming and renewing data on its recent sojourn assumptions or witnessing.

List of references

1. Солонніков В. Г. Оптимізація розподілу сил та засобів для виконання завдань при проведенні прикордонних (спільних) операцій / В. Г. Солонніков, І. С. Катеринчук // Труди академії. Випуск 61. — К. : НАО, 2005. — С. 18 — 24.

2. Катеринчук І. С. Математична модель визначення імовірних маршрутів руху порушників державного кордону / І. С. Катеринчук, С. М. Широбоков, М. Ю. Цибровський // Збірник наукових праць № 13. Частина І. — Хмельницький : Видавництво Національної академії Державної прикордонної служби України імені Б. Хмельницького, 2005. — С. 48 — 53.

3. Катеринчук І. С. Рекомендації щодо вибору раціональних режимів функціонування системи протидії незаконній міграції на державному кордоні / І. С. Катеринчук, А. Б. Мисик, М. Ю. Цибровський // Честь і закон. — № 1. — Х. : АВВ МВСУ, 2007. — С. 58 — 64.

4. Воробьёв Н. Н. Теория игр для экономистов-кибернетиков / Воробьёв Н. Н. — М. : Наука, Главная редакция физико-математической литературы, 1985. — 272 с.

5. Дьяконов В. Математические пакеты расширения MATLAB : [специальный справочник] / В. Дьяконов, В. Круглов. — СПб. : Питер, 2001. — 480 с. : ил.

6. Штовба С. Д. Проектирование нечетких систем средствами MATLAB / Штовба С. Д. — М. : Горячая линия — Телеком, 2007. — 288 с.

7. Романюк В. В. Метод реалізації оптимальних змішаних стратегій у матричній грі з порожньою множиною сідлових точок у чистих стратегіях з відомою кількістю партій гри / В. В. Романюк // Наукові вісті НТУУ “КПІ”. — 2009. — № 2. — С. 45 — 52.

8. Романюк В. В. Адаптація методу реалізації оптимальних змішаних стратегій у матричній грі з порожньою множиною ситуацій рівноваги з відомою наперед кількістю раундів гри у програмному середовищі MATLAB / В. В. Романюк // Вісник Хмельницького національного університету. Технічні науки. — 2009. — № 4. — С. 57 — 67.

Надійшла до редколегії 06.11.2009р. Рецензент: к.т.н., доц. Зеленьова І. Я.

В. В. Романюк
Хмельницький національний університет

Програмний засіб для практичної реалізації моделі пошуку активного об'єкта як багатоетапної діагональної гри. Представлено модель пошуку активного об'єкта усередині замкненої розділеної області як багатоетапну діагональну гру. Розв'язок цієї гри, що еволюціонує від етапу до етапу, може знаходитись за допомогою розробленого та представленого програмного засобу. Цей засіб повертає розв'язки та будує подібний до брусків розподіл їх елементів поряд з розподілом апріорних імовірностей перебування в області.
пошук об'єкта, діагональна гра, багатоетапна оптимізація, оптимальна поведінка, програмний Matlab-засіб

В. В. Романюк
Хмельницкий национальный университет

Программное средство для практической реализации модели поиска активного объекта как многоэтапной диагональной игры. Представлено модель поиска активного объекта внутри замкнутой разделённой области как многоэтапную диагональную игру. Эволюционирующее от этапа к этапу решение этой игры может находиться с помощью разработанного и представленного программного средства. Это средство возвращает решения и строит подобное брускам распределение их элементов рядом с распределением априорных вероятностей пребывания в области.
поиск объекта, диагональная игра, многоэтапная оптимизация, оптимальное поведение, программное Matlab-средство