Romanuke V.V.
Khmelnytskyy National University,
Khmelnytskyy, Ukraine

PREDICTING SURFACE WEAR NUMERICALLY ON WEAR STOCHASTIC PROCESS BUNDLE REALIZATIONS UNDER UNCERTAINTIES OF WEAR INTENSITY AND VOLATILITY IN ITO DIFFERENTIAL EQUATION

Wear prediction problem introduction

Predicting the solid material surface wear (SMSW) $w(t)$ through time $t \in [0; T]$ for the total exploitation expiration T opens a range of possibilities to control handling it. Also knowing the dynamics of wear allows evaluating the upper speed of the action over the material surface. And the wear dynamics law often appears a starting spindle for finding the function $w(t)$, though this law versions may be various, including uncertainty of their parameters [1], what bears deep complexities with trusting the function $w(t)$ shape eventually.

Wear prediction origins survey and the non-highlighted question emphasizing

There is a simplest SMSW prediction model [1] in the form of the Ito stochastic differential equation (ISDE) [2]

$$dw(t) = a(t)dt + \lambda(t)d\psi(t) \tag{1}$$

that could be stated also into the leftsided wear dynamics equation (WDE)

$$\frac{dw(t)}{dt} = a(t) + \lambda(t)\frac{d\psi(t)}{dt} \tag{2}$$

for the known SMSW intensity $a(t)$ and volatility $\lambda(t)$ up with the standard Wiener process $\{\psi(t)\}_{t\in[0;\,T]}$. This random process may be restated over the normally distributed variate Ξ with zero expectance and unit variance for its values $\{\xi(t)\}_{t\in[0;\,T]}$ as [2]

$$\psi(t) = \xi(t)\sqrt{t}\,. \tag{3}$$

Then with the known integral form solution (IFS)

$$w(t) = w(0) + \int_0^t a(\tau)d\tau + \int_0^t \lambda(\tau)d\psi(\tau) \tag{4}$$

of the ISDE (1) or WDE (2) at the given initial wear $w(0)$ there is a way to obtain the wear (3) numerically, substituting

$$d\psi(t) = \xi\sqrt{dt} \tag{5}$$

for the fixed random value ξ of the variate Ξ. Nevertheless, having the numerical solution of the ISDE (1) or WDE (2) may undergo demurring due to uncertainty of the function-parameters $a(t)$ and $\lambda(t)$, being stated more strictly like

$$a_{\text{low}}(t) \leqslant a(t) \leqslant a_{\text{up}}(t),\;\; \lambda_{\text{low}}(t) \leqslant \lambda(t) \leqslant \lambda_{\text{up}}(t). \tag{6}$$

So, the same lower and upper evaluations should be predicted and controlled for the wear IFS (4), although the wear lower evaluation is not as important as the upper.

Purpose for wear prediction as numerical IFS (4) of ISDE (1) under uncertainties (6)

In order to prevent false prediction of wear or its dynamics here is the paper purpose to find the lower and upper evaluations of wear as numerical IFS (4) of ISDE (1), driven with uncertainties (6) of SMSW intensity $a(t)$ and volatility $\lambda(t)$. This ought to be accomplished with modeling a bundle of wear stochastic processes (WSP) for every fixed $a(t)$ and $\lambda(t)$, changing them from their lower evaluations up to the upper ones. For that it is easily to apply the MATLAB environment, where it should be created a script-solver for ISDE (1) or WDE (2) as IFS (4), using (3) and (5) under uncertainties (6).

Predicting wear with a MATLAB script-solver for ISDE (1) as IFS (4) under uncertainties (6)

Within MATLAB environment one can freely model variate Ξ as many rounds, lengths and dimensions as needed. It is useful to normalize the total exploitation expiration T to the unit and then for samples

$$\left\{\xi_j = \xi\left(t_j\right)\right\}_{j=1}^{L} \quad \forall\, t_{i+1} = t_i + \frac{1}{L} \quad \text{for} \quad i = \overline{1,\, L-1} \quad \text{at} \quad t_1 = \frac{1}{L} \quad \text{and} \quad t_L = 1 \tag{7}$$

there is the numerical routine

$$w\left(t_j\right) = w(0) + \frac{1}{L}\sum_{i=1}^{j} a\left(t_i\right) + \sqrt{\frac{1}{L}}\sum_{i=1}^{j}\lambda\left(t_i\right)\xi_i \quad \text{for} \quad j = \overline{1,\, L} \tag{8}$$

of finding WSP (4) $\forall\, t \in [0;\, 1]$. Furthermore we may consider parameters $a(t)$ and $\lambda(t)$ constant $\forall\, t \in [0;\, 1]$ within the functional fields in (6) as the lower and upper functions in those fields are best accepted constant due to insignificant dynamics of the wear intensity and its volatility [1].

For making an example assume that

$$0.8 \leqslant a(t) \leqslant 1 \quad \text{and} \quad 0.079 \leqslant \lambda(t) \leqslant 0.081. \tag{9}$$

There is a MATLAB script-solver "wearinv" code on figure 1, which after having been run plots WSP bundle realizations (figure 2) as (8) on the background of the wear expectance

$$\mathbf{M}\left[w(t)\right] = w(0) + \int_0^t a(\tau)\,d\tau. \tag{10}$$

Here $w(0) = 0$ and $L = 1000$ by increments 0.01 and 0.001 for $a(t)$ and $\lambda(t)$ correspondingly.

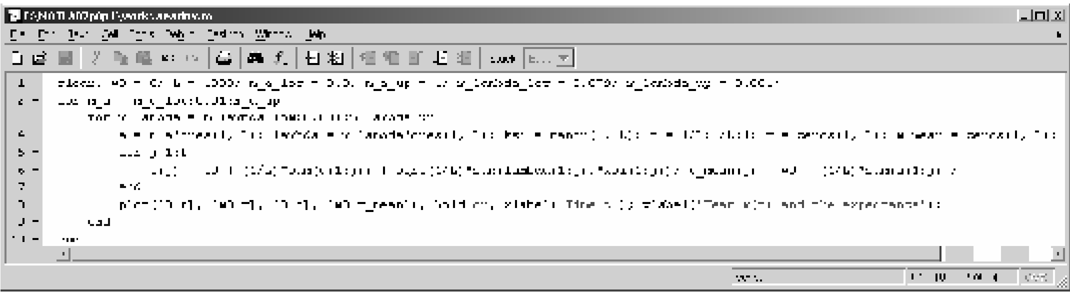

Figure 1 – MATLAB script-solver "wearinv" code for plotting WSP (4) bundle realizations by $w(0)=0$ and $L=1000$ for (9) at increments 0.01 and 0.001 for $a(t)$ and $\lambda(t)$ correspondingly

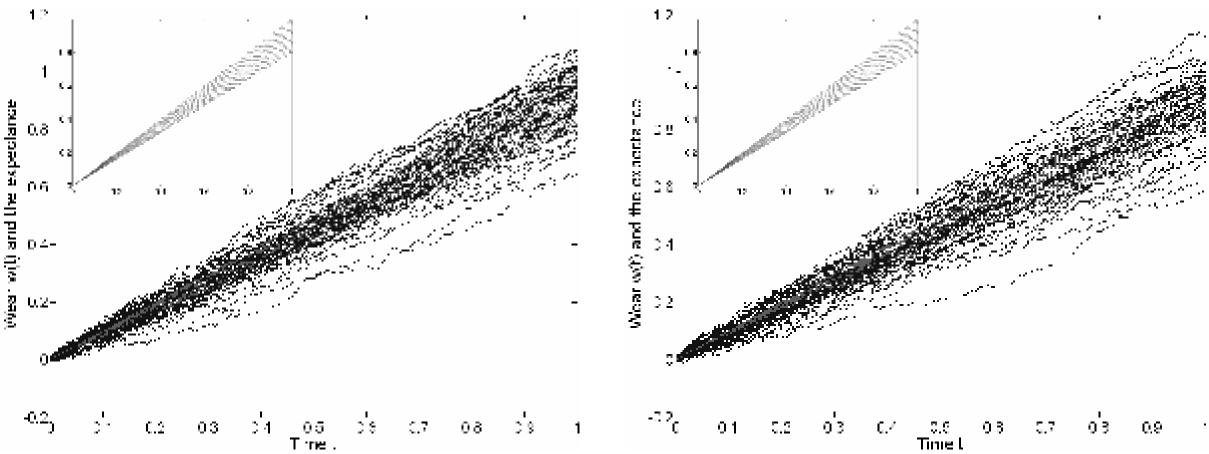

Figure 2 – WSP (4) bundle realizations, obtained twice by $w(0)=0$ and $L=1000$ for (9) at increments 0.01 and 0.001 for $a(t)$ and $\lambda(t)$ correspondingly on the background of 21 wear expectances bundle

Going for more complicated, for the case with functional uncertainty in SMSW intensity $a(t)$, which, say, lies within some zone, included functions

$$a(t) = \left(0.25b^3 + b\right)^{-1} \left(\left(bt - 0.5b\right)^3 + \left(bt - 0.5b\right)^2 + \left(bt - 0.5b\right) + 0.125b\left(b^2 - 2b + 4\right)\right) \qquad (11)$$

for $b = \overline{2, 12}$, with the same volatility uncertainty in (9) there is a MATLAB script-solver "wearinv2" code on figure 3, working analogously to "wearinv" (figure 4). Surely, codes of solver "wearinv" and solver "wearinv2" both may be easily modified for any needed cases with uncertainties in (6). Moreover, these MATLAB scripts may be used for creating advanced MATLAB scripts or modules, aimed for researching SMSW processes.

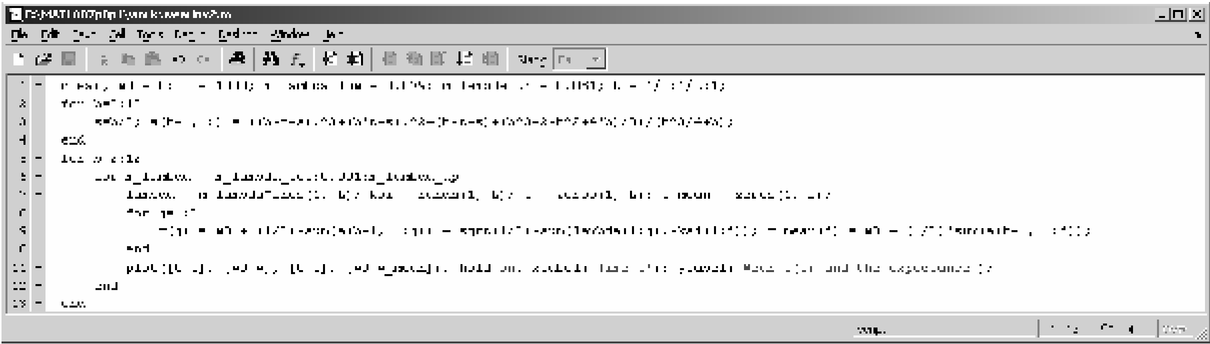

Figure 3 – MATLAB script-solver "wearinv2" code for plotting WSP (4) bundle realizations by $w(0) = 0$ and $L = 1000$ for the volatility uncertainty in (9) by the uncertainty in SMSW intensity $a(t)$ as (11) for $b = \overline{2, 12}$

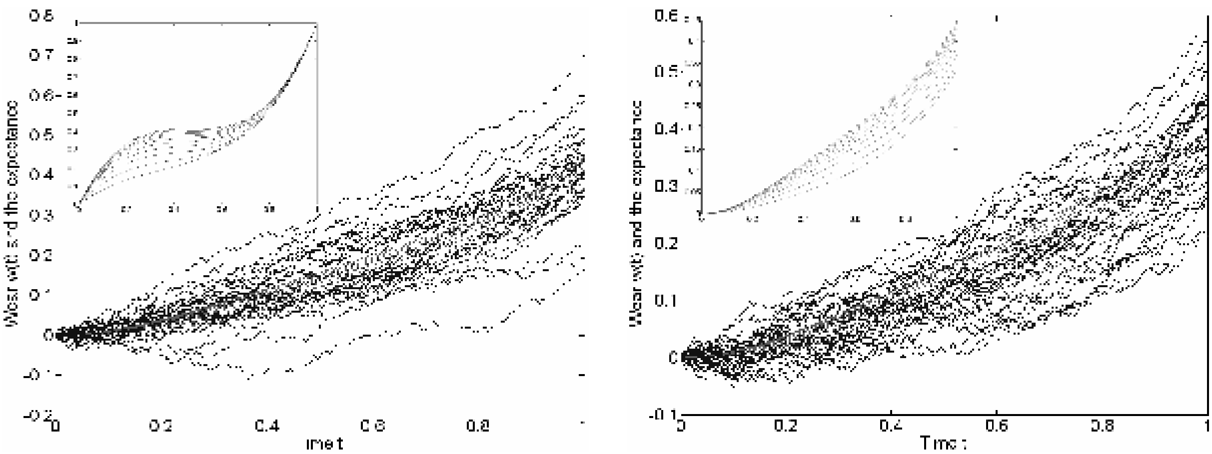

Figure 4 – Two WSP (4) bundle realizations by $w(0) = 0$ and $L = 1000$ for the volatility uncertainty in (9) by the uncertainty in SMSW intensity $a(t)$ as (11) for $b = \overline{2, 12}$ (on the left side) on the background of 11 wear expectances bundle (on the right side)

Conclusion and the further investigation outlook

Having visualized WSP (4) bundle realizations under corresponding conditions allows to watch its stretching width, though for predicting SMSW the most significant is the wear process upper evaluation (WPUE) in the bundle. This WPUE curve shows the worse case of wear could run. However, modeling bundles of WSP under uncertainties (6) should refer to the expectance (10), and with the bundle stretching more wide (through the time) it is recommended to inspect the working surface more scrupulously. And the further investigation outlook is on to schedule such inspections due to that WSP runs wider.

References

1. Andersson S. A random wear model for the interaction between a rough and a smooth surface / S. Andersson, A. Söderberg, U. Olofsson // Wear. – 2008. – Volume 264, Issues 9 – 10. – P. 763-769.

2. Кузнецов Д. Ф. Численное интегрирование стохастических дифференциальных уравнений : [монография] / Кузнецов Д. Ф. – СПб. : Изд-во С.-Петербургского государственного университета, 2001. – 712 с.

Надійшла 06.02.2012